Universidade Federal de Pernambuco

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Colóquio do DMat

On the well-posedness and stability analysis of standing waves for a 1D-Benney-Roskes system

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Abstract

In this talk, we revisit the well-posedness for the Benney-Roskes system (also known as Zakharov-Rubenchik systems) for $N = 1, 2, 3$, and establish the nonlinear orbital stability of standing waves (ground states) in $N = 1$,

$$\left\{\begin{array}{rl} i\partial_t\psi + \epsilon\partial_z^2\psi & = -\sigma_1\Delta_\perp\psi + (\sigma|\psi|^2 + W(\rho + D\partial_z\varphi))\,\psi, \\ \partial_t\rho + \sigma_2\partial_z\rho & = -\Delta_\perp\varphi - \partial_z^2\varphi - D\partial_z(|\psi|^2), \\ \partial_t\varphi + \sigma_2\partial_z\varphi & = -\frac{1}{M^2}\rho - |\psi|^2, \end{array}\right. \tag{1}$$

which describes the interaction of high-frequency and low-frequency waves in plasmas and magnetohydrodynamics, where we are using the notation $\mathbf{x} = (x, y, z)$ for $N = 3$, $\mathbf{x} = (x, z)$ for $N = 2$, $\Delta_\perp = \partial_x^2 + \partial_y^2$ for $N = 3$, and $\Delta_\perp = \partial_x^2$ for $N = 2$. The model was first derived for D. Benney and G. Roskes in the context of gravity waves (Beney-Roskes-1969) and also for A. Rubenchik and V. Zakharov in the context of the interaction of spectral narrow high frequency wave packet of small amplitude with low-frequency acoustic type oscillations (Rubenchik-Zakharov-1972).

In contrast with the existence results given by F. Oliveira for $N = 1$ (for a modified system) (Oliveira-2005)and G. Ponce and J-C. Saut for $N = 2, 3$ (Ponce-Saut-2005), we did not modify the Benney-Roskes system by taking the t-derivative in the last two nonlinear transport equations, reducing the system to a nonlinear Schrödinger equation coupled with two wave equations. Instated of this approach, we derive the whole group for the system and without modifying the variables, we proceed to analyze the well-posedness of the Cauchy problem. In the case $N = 1$, we prove that the standing waves of the Benney-Roskes system are orbitally stable, by using the Lyapunov method, which was used by M. Weinstein (Weinstein-1986) to establish stability for nonlinear Schrödinger equation (NLS) and the generalization of the Kortewegde Vries equation (GKdV).

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